arXiv:hep-th/0007003v1 1 Jul 2000

QUANTUM SPACE-TIME AND REFERENCE FRAMES IN GRAVITY AND FLAT SPACE-TIME

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Abstract

The quantum space-time model which accounts material Reference Frames (RF) quantum effects considered for flat space-time and ADM canonical gravity. As was shown by Aharonov for RF - free material object its c.m. nonrelativistic motion in vacuum described by Schrodinger wave packet evolution which modify space coordinate operator of test particle in this RF and changes its Heisenberg uncertainty relations. In the relativistic case we show that Lorentz transformations between two RFs include the quantum corrections for RFs momentum uncertainty and in general can be formulated as the quantum space-time transformations. As the result for moving RF its Lorentz time boost acquires quantum fluctuations which calculated solving relativistic Heisenberg equations for the quantum clocks models. It permits to calculate RF proper time for the arbitrary RF quantum motion including quantum gravity metrics fluctuations. Space-time structure of canonical Quantum Gravity and its observables evolution for RF proper time discussed in this quantum space-time transformations framework.

Submitted to Int. J. Theor. Phys.

Talk given at 'Quantum Field Theory under External conditions' conference , Leipzig, September 1998
to appear in proceedings.

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1 Introduction

The possible changes of space-time properties at small (Plank) scale now extensively discussed [1, 2]. Due to the absence of any experimental information it seems instructive to look for some directions exploring attentively the standard Quantum Physics space-time structure. Some years ago Aharonov and Kaufherr have shown that in nonrelativistic Quantum Mechanics (QM) the correct definition of physical reference frame (RF) must differ from commonly accepted one, which in fact was transferred copiously from Classical Physics [1]. The main reason is that to perform exact quantum description one should account the quantum properties not only of studied object, but also RF, despite the possible practical smallness. The most simple of this RF properties is the existence of Schroedinger wave packet of free macroscopic object with which RF is usually associated [3]. Then it introduces additional uncertainty into the measurement of object space coordinate in this RF. Furthermore this effect results in the states transformations between two such RFs which includes quantum corrections to the standard Galilean group transfromations [1]. Algebraic and group theorettical structure of this transformation was studied in [4]. In their work Aharonov and Kaufherr formulated Quantum Equivalence Principle (QEP) in nonrelativistic QM - all the laws of Physics are invariant under transformations between both classic and this finite mass RFs which called quantum RFs. The importance of RF quantum properties account was shown already in Quantum Gravity and Cosmology studies [5, 6, 7] and will be considered here in connection with the time problem in quantum gravity.

In this paper the consistent relativistic covariant theory of quantum RFs formulated, our preliminary results were published [8]. In this theory no new *ad hoc* hypothesis introduced; all calculations are performed in the standard QM formalism. It will be shown that the transformation of the particle state between two quantum RFs obeys to relativistic invariance principles, but differs from standard Poincare Group transformations, due to quantum relativistic correction for RF motion. Solving the evolution equation for quantum clocks models the proper time in moving quantum RF calculated and the related effects of RF momentum quantum fluctuations revealed. This clocks model applied for the analysis of the space-time structure of canonical quantum gravity [5].

In chap.2 canonical formalism for quantum RFs described. In chap.3 we study quantum clocks models and obtain relativistic proper time for quantum RF. In chap.4 the relativistic evolution equations and unitary transformations for quantum RFs described. We'll consider also RF quantum motion in gravitation field where gravitational 'red shift' results in additional clocks time fluctuations.

2 Quantum Coordinates Transformations

For the beginning we'll consider Quantum Measurements problems related to Quantum RFs model. In QM framework the system defined as RF should be able to measure the observables of studied quantum states and so include the measuring

device - detector D. As the realistic example we can regard the photoemulsion plate or the diamond crystal which can measure the particle position and simultaneously record it. Despite the multiple proposals up to now the established theory of collapse doesn't exist [9, 10]. Yet our problem premises doesn't connected directly with any state vector collapse mechanism and and it's enough to detailize standard QM collapse postulate of von Neumann measurement theory [10]. We consider RF which consists of finite number of atoms (usually rigidly connected) and have the finite mass. It's well known that the solution of Schroedinger equation for any free quantum system can be factorized as :

$$\Psi(t) = \sum c_l \Phi^c_l(\vec{R}_c, t) * \phi_l(u_k, t) \tag{1}$$

where center of mass coordinate $\vec{R}_c = \sum m_i * \vec{r}_i / M$, c_l are the partial amplitudes. u_k describes the internal degrees of freedom, which for potential forces are reduced to $\vec{r}_{i,j} = \vec{r}_i - \vec{r}_j$ [3]. Here Φ^c_l describes the c.m. motion of the system. It means that the evolution of the system is separated into the external evolution of pointlike particle M and the internal evolution defined by $\phi_l(u_k, t)$. So the internal evolution is independent of whether the system is localized in some 'absolute' reference frame (ARF) or not. Quantum Field Theory evidences that the factorization of c.m. and relative motion holds true even for nonpotential forces and variable N in the secondarily quantized systems [11]. Moreover this factorization expected to be correct for nonrelativistic systems where binding energy is much less then its mass M, which is characteristic for the real detectors and clocks. Consequently it's reasonable to assume that this factorization fulfilled also for the detector states despite we don't know their exact structure. For our problem it's enough to assume that eq.(1) holds for RF state only in the time interval T from RF preparation moment t_0 until the act of measurement starts , i.e. the measured particle n wave packet ψ_n impacts with D. If this factorization holds the space coordinate measured in this RF depends not only on ψ_n but also on Φ^c_l which permit in principle to study quantum RF effects. In this case the possible factorization violation at later time when the particle state collapse occured is unimportant for us. We regard in our model that all measurements are performed on the quantum pairs ensemble of particles G^2 and F^1. It means that each event is resulted from the interaction between the 'fresh' RF and particle ,prepared both in the specified quantum states, alike the particle alone in the standard experiment.

To illustrate the meaning of Quantum RF consider gedankenexperiment where in ARF the wave packet of RF F^1 described by $\psi_1 = \eta_1(x)\xi_1(y)\zeta_1(z)$ at time moment t_0. The test particle n with mass m_n belongs to narrow beam which average velocity is orthogonal to x axe and its wave function at t_0 is $\psi_n = \eta_n(x)\xi_n(y)\zeta_n(z)$. Before they start to interact this system wave function is the product of F^1 and n packets. We want to find n wave function for the observer in F^1 rest frame. In general it can be done by means of the canonical transformations described below, but if n beam state is localized so that ψ_n can be approximated by delta-function $\delta(x - x_b)\delta(y - y_b)\delta(z - z_b)$ then n wave function in F^1 can be easily calculated $\psi'_n(\vec{r}'_n) = \eta_1(x'_n - x_b)\xi_1(y'_n - y_b)\zeta_1(z'_n - z_b)$. It shows that if for example F^1 wave packet along x axe have average width σ_x then from the 'point of view' of observer in F^1 each

object localized in ARF acquires wave packet of the same width σ_x in F^1 and any measurement in F^1 and ARF will confirm this conclusion.

The generalized Jacoby canonical formalism will be applied in our model alternatively to Quantum Potentials used in [1]. Consider the system S_N of N objects W^k which include N_f frames F^i which have also some internal degrees of freedom and $N_g = N - N_f$ pointlike 'particles' G^i. At this stage we can regard both of them as equivalent objects in the relation to their c.m. motion. We'll assume for the beginning that particles and RFs canonical operators $\vec{r}_i, \vec{p}_i$ are defined in absolute (classical) ARF - F^0 having very large mass m_0, but later this assumption can be abandoned. We'll start with Jacoby canonical coordinates $\vec{u}^l_j$ associated with F^l rest frame, which for $l = 1$ equal :

$$\vec{u}^1_i = \frac{\sum\limits_{j=i+1}^{N} m_j \vec{r}_j}{M_{i+1}} - \vec{r}^l_i; \quad 1 \leq i < N; \quad \vec{u}_N = \vec{u}_s = \vec{R}_c \tag{2}$$

where $M_i = \sum\limits_{j=i}^{N} m_j$. $\vec{u}^l_i$ can be obtained and is the linear combination of $\vec{u}^1_i$. Conjugated to $\vec{u}^l_i$ canonical momentums are :

$$\vec{\pi}^1_i = \mu_i \left(\frac{\vec{p}^s_{i+1}}{M_{i+1}} - \frac{\vec{p}_i}{m_i} \right), \quad \vec{\pi}_N = \vec{p}_s = \vec{p}^s_1 \tag{3}$$

where $\vec{p}^s_i = \sum\limits_{j=i}^{N} \vec{p}_j$,and reduced mass $\mu_i^{-1} = M_{i+1}^{-1} + m_i^{-1}$. The relative coordinates $\vec{r}_j - \vec{r}_1$ can be represented as the linear sum of $\vec{u}^1_i$. They don't constitute canonical set due to the quantum motion of F^1 [1]. The Hamiltonian of S_N motion in ARF is expressed also via momentums $\vec{\pi}^1_i$:

$$\hat{H} = \sum_{i=1}^{N} \frac{\vec{p}_i^2}{2m_i} = \frac{\vec{p}_s^2}{2M} + \sum_{j=1}^{N-1} \frac{(\vec{\pi}^1_j)^2}{2\mu_j} = \hat{H}_s + \hat{H}_c \tag{4}$$

In F^1 rest frame the true observables are $\vec{\pi}^1_i, \vec{u}^1_i$ and it's impossible to measure S_N observables $\vec{p}_s$ and $\vec{R}_c$. The true Hamiltonian of S_N in F^1 should depend on the true observables only , so we can regard $\hat{H}_c$ as the real candidate for its role. It results into modified Schroedinger equation which depends not only of particles masses ,but on observer mass m_1 also.

Now we'll regard here the alternatve form of this formalism which use Jacoby frame condition (JFC) and is more convenient for the relativistic problem. For the described system S_N Langrangian in ARF $L = \sum \frac{m_i \dot{\vec{r}}_i^2}{2}$ gives H of (4) after Legendre transform. If one wish to include ARF motion in this formalism the simplest way is to define formally $L' = L + \frac{m_0 \dot{\vec{r}}_0}{2}$. It gives $N+1$ canonical momentums : $\vec{p}_j = \frac{\partial L'}{\partial \dot{\vec{r}}_j}$. The new Langrangian L' is formally symmetric relative to the frame choice and it gives the Hamiltonian $H' = H + \frac{\vec{p}_0^2}{2m_0}$ for H of (4). Due to it to anchor this momentums and H' to F^i rest frame in which they acquire some values one must broke L' symmetry

introducing the frame condition (FC) or kinematical (holonomial) constraint [13]. For ARF rest frame we choose FC $\vec{p}_0^{2n} \approx 0$, where from the formal reasons $n = 2$. It means that $\dot{\vec{r}}_0 = 0$ - RF is at rest relative to itself which seems quite natural, yet it differs from FC used in [1]. All Classical and QM results are reproduced in this scheme if ARF mass is taken infinite. S_N quantization in F^1 performed with Hamitonian $\hat{H}'$ and FC regarded as the operator which obeys to Dirack rules for the first order constraints [12, 13]. Galilean-like passive transformations from ARF to F^1 and back can be found introducing FC also for F^1 $\vec{p}_{11}^{2n} \approx 0$, where $\vec{p}_{1i}$ are the canonical momentums in F^1. S_{N+1} unitary transformation from ARF to F^1 is convenient to write via the $F^{0,1}$ total momentum $\vec{p}_f = \vec{p}_0 + \vec{p}_1$ and F^0, F^1 relative momentum $\vec{\pi}_f$ conserving other momentums $\vec{p}_i$. Their conjugated coordinates $\vec{r}_f, \vec{u}_f$ have the standard form of (2). In this notations the transformation from F^0 to F^1 is equal to :

$$U_{1,0} = P_f e^{i a_f \vec{p}_f \vec{r}_f} e^{-i\vec{p}_f \vec{b}_s} \prod_{i=2}^{N} e^{-i m_i \vec{r}_i \vec{\beta}} \tag{5}$$

where $a_f = \ln \frac{m_0}{m_1}, \vec{b}_s = \frac{M}{m_0}\langle \vec{R}_c \rangle$. P_f is $\vec{r}_f$ reflection (parity) operator. $\vec{\beta} = \frac{\vec{p}_f}{m_1}$ is the operator corresponding to the velocity parameter in Galilean transformation. Under this transformation $\vec{p}_f$ transformed to $\vec{p}_{1f} = \vec{p}_{10} + \vec{p}_{11}$ and $\vec{\pi}_{1f} = \vec{\pi}_f$. Alike the transformation from $\vec{p}_j$ to $\vec{\pi}_i^1$ obtained operator $U_{1,0}$ includes the dilatation transformation [14].

For $N = 2$ one obtains F^1 momentums and coordinates :

$$\begin{aligned}
\vec{p}_{10} = (1 - \frac{m_0}{m_1})\vec{p}_0 - \frac{m_0}{m_1}\vec{p}_1 \quad ; \quad \vec{r}_{10} = -\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_0} + \vec{b}_s \\
\vec{p}_{11} = -\vec{p}_0 \quad ; \quad \vec{r}_{11} = -\vec{r}_0 + (1 - \frac{m_1}{m_0})\vec{r}_1 - \frac{m_2}{m_0}\vec{r}_2 + \vec{b}_s \\
\vec{p}_{12} = -\frac{m_2}{m_1}(\vec{p}_0 + \vec{p}_1) + \vec{p}_2 \quad ; \quad \vec{r}_{12} = \vec{r}_2
\end{aligned} \tag{6}$$

Results for $N > 2$ can be easily deduced from this formulaes. It's easy to see that ARF FC transformed into F^1 FC. All $\vec{\pi}_i^1$ are conserved and space shift on $\vec{b}_s$ conserves all the distances $\vec{r}_i - \vec{r}_j$. In the limit where heavy F^1 moves nearly classically U_{01} becomes the Galilean momentum transformation with the velocity $\langle \vec{\beta} \rangle$. S_{N+1} Hamitonian in F^1 also can be rewritten via new relative momentums $\vec{\pi}_{1j}$ which can be easily derived following (3) :

$$\hat{H}^1 = \sum_{i=0}^{N} \frac{\vec{p}_{1i}^2}{2m_i} = H_s^1 + H_c^1 = \frac{\vec{p}_{1s}^2}{2M_{N+1}} + \sum_{j=2}^{N+1} \frac{\vec{\pi}_{1j}^2}{2\mu_{1j}} \tag{7}$$

The term $\hat{H}_s^1$ describes S_N c.m. motion relative to F^1 which doesn't influence on the evolution of S_{N+1} true observables $\vec{\pi}_{1i}, \vec{u}_{1i}$ or $\vec{r}_i - \vec{r}_j$. $\vec{r}_{1i}, \vec{p}_{1i}$ aren't S_{N+1} observables for F^1 observer, yet $\vec{p}_{1i}$ expectation values can be found from $\vec{\pi}_i^1$ measurements.

Now we have quantum system S_{N+1} which include ARF and in ARF rest frame we can ascribe to it without any contradictions with QM the state vector which for $N = 2$ is equal : $\psi_s(\vec{p}_0, \vec{p}_1, \vec{p}_2) = \varphi(\vec{p}_1, \vec{p}_2)|\vec{p}_0 = 0\rangle|\vec{p}_1\rangle|\vec{p}_2\rangle$. After $U_{1,0}$ transformation

it acquires the similar form in F^1 rest frame with $|\vec{p}_{11}=0\rangle$. As the result of this transform we obtain the new canonical coordinates referred to finite mass F^1 rest frame. They permit to factorize internal S_N motion and ARF motion and dropping ARF term in H^1 of (7) we obtain S_N Hamitonian. Remind that active transformation shifts G^2 state ψ_2 on the distance $\vec{a}$ and velocity $\vec{\beta}$ relative to RF. Passive G^2 transformation means the transition from one RF to another, but for quantum RF with state ψ_s it can't be described by any state shift on $\vec{a},\vec{\beta}$ and have more complicated form. $U_{1,0}$ is such passive transformation and active G^2 transformation is the standard Galilean one even in F^1 [11].

In general the quantum transformations in 2 or 3 dimensions should also take into account the possible rotation of quantum RF axes relative to ARF, which introduce additional angular uncertainty into objects coordinates. Thus after performing coordinate transformation $\hat{U}_{A,1}$ from ARF to F^1 c.m. we must rotate all the objects (including ARF) around it on the uncertain polar and azimuthal angles , ϕ_1, θ_1 which are F^1 internal degrees of freedom. We can imagine F^1 axes as some solid rods which orientation this angles describe. As the result the complete transformation is: $\hat{U}^T_{A,1}=\hat{U}^R_{A,1}\hat{U}_{A,1}$. Such rotation transformation operator commutes with $\hat{H}_c$ and due to it can't change the evolution of the transformed states [1, 8].

3 Quantum Clocks Models

To construct the relativistic covariant formalism of quantum RFs it's necessary first to define the time in such RFs. In nonrelativistic mechanics time t is universal and is independent of observer, while in relativistic case each observer in principle has its own proper time τ. We don't know yet the nature of time , but phenomenologically it can be associated with the clock hands motion or some other relative motion of the system parts [17]. In Special Relativity the time in moving frame F^1 can be defined by external observer at rest measuring the state of F^1 comoving clocks. We'll consider the same procedure in relativistic QM i.e. some clock observable being measured at some time from the rest frame gives the estimate of proper time of moving quantum RF F^1.

For some clocks models F^1 internal evolution which define F^1 clocks motion and consequently its proper time τ_1 can be factorized from F^1 c.m. motion. Its quantum c.m. motion described by the relativistic Schrodinger equation for massive boson. This is Klein-Gordon square root (KGR) equation in which only positive root will be regarded for initial positive energy state [11]. Solving Dirack constraints it was shown recently that this first order equation is completely equivalent to free Field secondary quantization [15].

For our relativistic model we should regard more strictly the features of reference frames and clocks, taking into account the internal motion. Consider the evolution of some system F^1 where the internal interactions described by the Hamiltonian $\hat{H}_c$ are nonrelativistic , which as was discussed in chap.1 is a reasonable approximation for the measuring devices or clocks. We'll use the parameter $\alpha_I=\frac{\bar{H}_c}{m_1}$,where m_1 is F^1 constituents total rest mass. In F^1 c.m. $\alpha_I=m_1^{-1}\langle\varphi_c|\hat{H}_c|\varphi_c\rangle$ where φ_c is F^1

internal state of (1). It describes the relative strength of the internal F^1 interactions and for the realistic clocks is of the order 10^{-10}. In addition we'll assume that all RF constituents spins and orbital momentums are compensated so that its total orbital momentum is zero, like in α-particle ground state. In this case the system F^1 c.m. motion can be reduced to the motion of the spinless boson with the mass m_1 and in the next order the mass operator $m_t = m_1 + H_c$ will be used. We'll start the proper time study with the simple models of quantum RFs with clocks, yet we expect its main results to be true also for the more sophisticated models.

To introduce our main idea let's regard the dynamics of the moving clocks in Special relativity [7]. We'll suppose that the proper (clocks) time is defined by the coordinate θ describing some internal system motion independent of its c.m. motion. For the simplicity assume that Hamiltonian of clocks H_c results in the trajectory $\theta(t) = \omega t + \theta_0$ of the clocks canonical observable θ, which renormalized into the time observable $\tau = \frac{\theta}{\omega}$. This is the property which is expected from ideal clocks and the simplest example of such system is the motion of free particle relative to observer $\tau = \frac{x}{v}$ [17]. For this and some other clocks models described below the Hamitonian of clocks with mass m_1 which c.m. moves with momentum $\vec{p}_1$ relative to ARF :

$$H_T = (m_t^2 + \vec{p}_1^2)^{\frac{1}{2}}$$

where $m_t = m_1 + H_c$. If $\theta, \vec{p}_1$ commutes, solving Hamilton equations in ARF time τ_0 one obtains $\theta(\tau_0) = B_1 \omega \tau_0 + \theta_0'$, where $B_1 = \frac{m_t}{H_T}$ coincides with Lorentz boost value. So as expected , if θ is measured by the observer at rest he finds the proper time $\tau_1 = B_1 \tau_0$ of moving frame. Yet we'll show that the quantum fluctuations of RF motion results in the principally new additional effects.

One of the most simple and illustrative quantum clocks models is the quantum rotator proposed by Peres [16]. The rotator Hamiltonian $\hat{H}_c = -2\pi\omega i \frac{\partial}{\partial\theta}$, where θ is the rotator's polar angle. Preparing the special initial state $\varphi_c(\theta) = |v_J^0\rangle$ at $t = 0$, where J is its maximum orbital momentum one obtains the close resemblance of the classical clocks hand motion. The clocks state $\varphi_c(\theta - 2\pi\omega t)$ for large J has the sharp peak at $\bar{\theta} = 2\pi\omega t$ with the uncertainty $\Delta_\theta = \pm\frac{\pi}{N}$ and can be visualized as the constant hand motion on the clocks circle.

Our main clocks model - C_x exploits the nonrelativistic particle motion relative to observer with Hamitonian $H_c = \frac{\vec{p}^2}{2m}$ [17]. Let's consider the particle 3-dimensional motion, but choose as its initial state at $t = 0$ the Gaussian packet factorized in x direction which momentum state vector is :

$$\phi_c(\vec{p}) = A\phi(p_y, p_z) e^{\frac{\sigma_x^2}{2}(\bar{p}_x - p_x)^2} \tag{8}$$

for which $\bar{p}_x \neq 0$. σ_x is the initial wave packet spatial spread. Then the simplest Hermitian observable which gives the time estimate is $\hat{\tau} = \frac{mx}{\bar{p}_x}$ - the particle's position on the arbitrary x axe. It describes the nonshifted measurement with $\bar{\tau} = t$ and the finite dispersion $D_0(t)$ for $0 < t < \infty$[17]. In fact in C_x model $\hat{\tau}$ is the clocks hand position operator or the pseudotime operator, and not a time operator in a strict sense [17, 16]. So from all sides C_x can be regarded as the realistic clocks model in which measuring $\hat{\tau}$ one obtains the correct t estimate with some statistical

error having quantum origin. C_x wave function $\varphi_c(x,t)$ evolution can be factorized as the packet centre of gravity motion with the constant velocity $\frac{p_x}{m}$ and the packet smearing around it. For the given initial state there is unambiguous correspondence between the state vector $|\varphi_c(x,t)\rangle$ and time t, so the quantum clocks synchronization at $t = 0$ means the preparation of the state $\varphi_c(x,0)$. From the corresponding Heisenberg equation one can find Heisenberg position operator for the Hamiltonian H_c :

$$x(t) = (\frac{p_x t}{m} + x_0) \tag{9}$$

where $x_0 = x(0)$ is Schrodinger position operator If $\bar{x}_0 = 0$ the corresponding clock time operator, which will be extensively used in relativistic theory can be decomposed as :

$$\hat{\tau} = t + \frac{p_x - \bar{p}_x + x_0 m}{\bar{p}_x}$$

The first term gives the time expectation value and the rest gives the clocks dispersion $D(t)$. To simplify our discussion we'll consider also the clocks model C_0 with the linear approximation of the position operator $x(t) = \omega t + x_0$ where parameter $\omega = \frac{\bar{p}_x}{m}$ which is the analog of Peres clocks for unbounded motion. C_0 Hamiltonian $H_c^0 = \omega p_x$ is unbounded from below for the continuous spectra, but for the interpretation of the relativistic clock effects it's unimportant. Any initial C_0 state (8) evolves as $\varphi_c^0(x - \omega t)$, so the initial form of wave function is conserved and only its centre of gravity moves.

Now we'll consider the relativistic C_x model in which RF F^1 and the particle G^2 system S_2 motion is relativistic. We'll suppose that ARF proper time τ_0 is defined also by some quantum clocks ,which dispersion is so small that can be neglected and τ_0 is the parameter. If F^1 internal interactions neglected F^1 c.m. motion described by the massive boson wave packet evolution and S_2 Hamiltonian H_T in ARF is the sum of two KGR Hamiltonians for the positive energy states [11, 15]:

$$H_T = (m_1^2 + \vec{p}_1^{\,2})^{\frac{1}{2}} + (m_2^2 + \vec{p}_2^{\,2})^{\frac{1}{2}} = (s + \vec{p}_s^{\,2})^{\frac{1}{2}} \tag{10}$$

, where $\vec{p}_s = \vec{p}_1 + \vec{p}_2$ and s is invariant mass square. $\sqrt{s}$ can be regarded as the Hamiltonian of two objects G^2, F^1 relative motion in their c.m.s. equal to system S_2 mass operator :

$$m_t = \sqrt{s} = (m_1^2 + \vec{q}^{\,2})^{\frac{1}{2}} + (m_2^2 + \vec{q}^{\,2})^{\frac{1}{2}} \tag{11}$$

where $\vec{q}$ is G^2 relative invariant momentum [21]. If $|\bar{q}|$ is small we can choose as p_x - clock momentum $\vec{q}$ projection along any suitable direction for which $\bar{q}_x \neq 0$. In this case F^1, G^2 relative motion can be regarded as nonrelativistic and F^1 mass operator approximated :

$$m_t \simeq m_s + \frac{q_x^2}{2\mu_{12}} + E_k(q_y, q_z)$$

,where μ_{12} is G^1, F^2 reduced mass, $m_s = m_1 + m_2$ is S_2 rest mass. In this case E_k is small and can be omitted in the calculations. Like in nonrelativistic case F^1 proper time in this C_x relativistic model can be estimated measuring in ARF the distance

$x = x_2 - x_1$ between F^1 and the particle G^2 which operator is equal to : $x = i\frac{\partial}{\partial q_x}$. For the obtained m_t S_2 Hamiltonian H_T can be formally rewritten :

$$H_T = [(m_s + H_c)^2 + \vec{p}_s^{\,2}]^{\frac{1}{2}} \tag{12}$$

where $H_c = \frac{q_x^2}{2\mu_{12}}$. Moreover it is reasonable to assume that this square root Hamitonian can describe the evolution of any clocks model with nonrelativistic interactions H_c i.e. for $\alpha_I \ll 1$ [11]. Here and below the algebraic operations with the operators (if they don't result into singularities) means Tailor raw decomposition. If F^1, G^2 relative motion is nonrelativistic we can assume for the beginning that F^1 and S_2 c.m.s. proper time practically coincide. For the classical motion F^1 Lorentz factor in S_2 c.m.s. $(1+\frac{\vec{q}^{\,2}}{m^2})^{\frac{1}{2}}$ and below we'll show that in quantum case their difference is also negligible. It's impossible to resolve in analytical form the Schrodinger equation for H_T of (12) , only some approximated solutions discussed below can be found. S_2 observables evolution can be found solving Heisenberg equation for the Hamitonian H_T of (12) or for exact Hamitonian of (10) as will be done below [17]. After the simple algebra one obtains x evolution in ARF proper time τ_0 :

$$\dot{x} = -i[x, H_T] = \frac{-im_t}{(m_t^2 + \vec{p}_s^{\,2})^{\frac{1}{2}}}[x, H_c] = -iB_1[x, H_c] \tag{13}$$

We'll call the operator $B_1(\vec{p}_s, m_t)$ the time boost operator, which interpretation will be discussed after some calculations. The clock observables we obtain in this clock models are the functions of canonical momentums only and due to it their factor ordering is unimportant for our problem. After the commutators calculations we can approximate operator m_t by the parameter $m_t \simeq m_s + \frac{\bar{q}_x^2}{2m}$. The operator x easily restored from $\dot{x}$:

$$x(\tau_0) = B_1(\vec{p}_s, m_t)\frac{q_x \tau_0}{\mu_{12}} + x_0$$

where x_0 is Schroedinger position operator for $\tau_0 = 0$. If we take that $\bar{x}_0 = 0$ it results into F^1 proper time operator :

$$\hat{\tau}_1 = B_1(\vec{p}_s, m_t)\frac{q_x}{\bar{q}_x}\tau_0 + \frac{\mu_{12} x_0}{\bar{q}_x} \tag{14}$$

Its meaning will be discussed after some calculations, but formally it's F^1 moving clocks hand position measured in ARF at the moment τ_0. τ_1 operator in C_0 model have the simpler form which prompts its interpretation :

$$\hat{\tau}_1 = B_1(\vec{p}_s, m_t)\tau_0 + \frac{x_0'}{\omega} \tag{15}$$

If $\bar{x}_0' = 0$ C_0 $\hat{\tau}_1$ expectation value $\bar{\tau}_1 = \bar{B}_1\tau_0$ coincides with the classical Lorentz time boost value. Its dispersion have the form :

$$D_\tau = D_L(\tau_0) + D_c = D_B\tau_0^2 + \bar{D}_2\tau_0 + D_0 \tag{16}$$

where $D_B = \bar{B}_1^2 - (\bar{B}_1)^2$ and $D_0 = \langle \frac{x_0'^2}{\omega^2} \rangle$ is the clocks mechanism dispersion, which for C_0 is time independent. Operator D_2 is equal to :

$$D_2 = \frac{B_1 x_0 + x_0 B_1}{\omega} \tag{17}$$

The numerical calculations show that for C_0 localized states D_2 expectation value is very small and can be neglected. If D_0 is small τ_1 fluctuations are defined mainly by $D_L(\tau_0)$ Lorentz boost dispersion stipulated by $\vec{p}_s$ fluctuations in F^1 wave packet. It's independent of the clocks mechanism and demonstrates that the proper time measurement have the principal quantum uncertainty growing unrestrictedly proportional to τ_0^2.

For C_x model the factor $\frac{q_x}{\bar{q}_x}$ in (14) produces additional $\hat{\tau}_1$ fluctuations. Due to it Lorentz boost expectation value differs only for the small factor of the order α_I :

$$\bar{\tau}_1 = \tau_0 \bar{B}_1 [1 + \frac{\bar{B}_1}{\sigma_x^2 \mu_{12} m_s}(1 - \bar{B}_1^2)]$$

It results from m_t dependence on p_x and reflects influence of clocks energy on total mass. We'll neglect this effect in C_x dispersion also described by ansatz (16), but with different parameters :

$$D_2 = \frac{\mu_{12}}{\bar{q}_x^2}(q_x B_1 x_0 + x_0 q_x B_1); \tag{18}$$

$$D_B = \frac{\bar{q}_x^2}{(\bar{q}_x)^2}\bar{B}_1^2 - (\bar{B}_1)^2; \quad D_0 = \frac{\mu_{12}^2 \sigma_x^2}{\bar{q}_x^2} \tag{19}$$

Here $\bar{D}_2 = 0$ for the gaussian wave packets (8) and any other localizable states. Due to q_x fluctuations absent in C_0 model the part of $D(\tau_1)$:

$$D_x = D_0 + \frac{\bar{q}_x^2 - (\bar{q}_x)^2}{(\bar{q}_x)^2}(\bar{B}_1)^2 \tau_0^2$$

can be related to the packet smearing along x coordinate, regarded as the clocks mechanism uncertainty.

To illustrate the physical meaning of this time operator let's consider the corresponding approximate solutions of F^1 state evolution equation for Hamiltonian (12). For $\alpha_I \to 0$ we can decompose H_T of (12) in the first α_I order :

$$-i\frac{d\Psi_s}{d\tau_0} \simeq [(m_s^2 + \vec{p}_s^2)^{\frac{1}{2}} + \frac{m_1 \hat{H}_c}{(m_s^2 + \vec{p}_s^2)^{\frac{1}{2}}}]\Psi_s \tag{20}$$

Here the first term is independent of H_c which permit to represent Ψ_s as the sum of factorized states. The second term is in fact the product of clock Hamitonian and Lorentz boost B_1. Let's choose the initial F^1 state $\Psi_s(0) = \Phi_s(\vec{p}_s)\varphi_c(x, 0)$ and $\Phi_s = \sum c_l |\vec{p}_{sl}\rangle$, where the sum denotes the integral over $\vec{p}_s$. From our definition of quantum clocks synchronization it follows that $\Psi_s(0)$ describes F^1 clocks synchronized with ARF clocks at $\tau_0 = 0$. Solving equation (20) one finds :

$$\Psi_s(\tau_0) = \sum c_l \varphi_c(x, B_l \tau_0)|\vec{p}_{sl}\rangle e^{-iE(\vec{p}_{sl})\tau_0} \tag{21}$$

where $E(\vec{p}) = (m_s^2 + \vec{p}^2)^{\frac{1}{2}}$, $B_l = B_1(\vec{p}_{sl}, m_s)$. For linear clock C_0 Hamiltonian $H_c = H_c^0$ for small α_I this state can be rewritten :

$$\Psi_s(\tau_0) = \sum_l c_l \varphi_c^0(x - \omega B_l \tau_0) \; |\vec{p}_{sl}\rangle e^{-iE(\vec{p}_{sl})\tau_0} \tag{22}$$

To make the situation more clear suppose that $\varphi_c^0(0) = \delta(x)$, which evolves at rest into $\delta(x - \omega\tau_0)$. Then x measurement defines the time τ of quantum clocks at rest unambiguously and with zero dispersion, but Ψ_s of (22) in general isn't x eigenstate. It means that at any $\tau_0 > 0$ Ψ_s is the entangled superposition of the states φ_c^0 which F^1 clocks acquires at the consequent τ_1 moments. As was shown there is one-to one correspondence between clock state $\varphi_c(x, t)$ and the time moment t and in some sense it can be regarded as the 'superposition' of F^1 proper time moments, or more precisely F^1 states existed at this moments. For example F^1 clocks hand can show 3,4 and 5 o'clocks simultaneously which can be tested by x measurement at some τ_0 in ARF. This spread corresponds to D_B dispersion term resulting from the F^1 momentum $\vec{p}_s$ uncertainty. For the realistic clocks their x dispersion given by D_0 isn't zero even at rest and this two terms added as statistically independent effects. Ψ_s for C_x Hamitonian is given by (21) and admits the same interpretation. It corresponds to the more complicated form of time dependent dispersion (19) which can be eventually factorized into the same two parts - relativistic and clock mechanism. So we conclude that the interpretation which follows from the approximate Schrodinger equation agrees well with Heisenberg operator calculus. In fact operator τ_1 describes F^1 proper time in the limit when this clock dispersion is very small and the clock energy is much less then F^1 total mass energy i.e. $\alpha_I \to 0$.

Obtained results suppose that the proper time of any quantum RF being the parameter in it simultaneously will be the operator from the 'point of view' of other RF. Qualitatively the appearance of RF proper time fluctuations can be understood considering the superposition of momentum eigenstates $|\vec{p}_{si}\rangle$ in S_2 wave packet as the superposition of S_2 velocities $\vec{\beta}_i$ and corresponding Lorentz factors $\gamma_1(\vec{\beta}_i)$. In Special Relativity F^1 proper time τ_1 measured at the same τ_0 in ARF depends on γ_1. If we formally extends this dependence on F^1 wave packet motion we get that the proper time will fluctuate proportionally to γ_1 spread. So F^1 clocks measurement in ARF shows how much time passed in F^1 in this particular event and can give the different value for another event of the same ensemble. It means that the time moments in different RFs corresponds only statistically with the dispersion D_τ in ARF given by (16). It differs from Special Relativity where one to one correspondence between τ_1, τ_0 time moments always exists , but can be incorporated into relativistic QEP if we find the analogous time relations between two quantum RFs of finite mass.

In fact τ_1 is more correct to relate to S_2 c.m.s. rest frame, but regarding the difference between F^1 and S_2 c.m.s. proper time operators τ_1', τ_1 it's easy to show that they coincide if $\bar{q}_x \to 0$. From it we conclude that the principal part of the relativistic time operator, independent of any particular clocks mechanism features have the form in the limit $\alpha_I \to 0$:

$$\hat{\tau}_1' = B_1(\vec{p}_1, m_1)\tau_0 \tag{23}$$

Moreover this formulae permits to define formally the time operator for any object including the single massive particle. This operator form of τ'_1 is closely connected with Fock-Shwinger proper time τ_F formalism interpretation and will be discussed in detail in the forcoming paper [18, 11]. Note only that $\hat{\tau}'_1(\tau_0)$ measurement gives F^1 proper time τ_F estimate at τ_0 moment of ARF time. On the opposite in Fock-Shwinger formalism τ_F is the parameter time to which particular values operators $\hat{\tau}_0(\tau_F), \vec{r}_1(\tau_F)$ related. In distinction with our formalism it makes τ_F interpretation confusing, because $\vec{r}_1$ and other F^1 operators are measured in ARF, hence the time of measurement defined in F^1 to which as we have shown in quantum case they related only statistically.

The practical realization of x measurement in ARF can be the intricated procedure, which scheme we don't intend to discuss here. Note only that to perform it one should measure simultaneously the distance between F^1 and G^2 and their total momentum giving total velocity and this two operators commute. Some examples of the analogous nonlocal observables measurements are described in [20]. The most disputable question here is the relativistic particle coordinate measurements. Yet in the considered case, when the relative F^1, G^2 average velocity is small then x is the nonrelativistic coordinate operator. Yet to prove the quantum equivalence principle it's necessary to perform the full relativistic calculations. We'll present such completely relativistic results for C_x model using Newton-Wigner Hermitian operator of the space coordinate [19] which is the direct analog of nonrelativistic operator x_1 :

$$\hat{x}^1_{NW} = i\frac{d}{dp_{x1}} - i\frac{p_{x1}}{2(m_1^2 + \vec{p}_1^2)} \tag{24}$$

The operator of two objects relative coordinates conjugated to c.m. momentum q_x can be derived from this objects c.m. Hamiltonian (11) :

$$\hat{x}_{NW} = x + F(\vec{q}) = i\frac{d}{dq_x} - i\frac{q_x}{\sqrt{s}}(\frac{1}{w_1} + \frac{1}{w_2}) \tag{25}$$

where $w_i = (m_i^2 + \vec{q}^2)^{\frac{1}{2}}$. The clocks time observable in F^1 rest frame is proportional to x_{NW} :

$$\tau = \frac{x_{NW} - \bar{x}_{NW}(0)}{\bar{\beta}_x}$$

where $\beta_x = q_x(w_1^{-1} + w_2^{-1})$ is F^2, G^1 relative velocity,

If we choose $\bar{x}_{NW}(0) = 0$, then solving Heisenberg equation in ARF for the Hamiltonian of (10) we find the resulting F^1 time operator :

$$\hat{\tau}_1 = \frac{B_1(\vec{p}_s, m_t)\tau_0\beta_x + x_{NW}(0)}{\bar{\beta}_x} \tag{26}$$

where in B_1 $m_t = \sqrt{s}$. This is the exact relativistic expression for τ_1 without assumption of q_x smallness. $\bar{\tau}_1$ corresponds to Lorentz boost value $\bar{B}_1$ which depends both on $\langle\vec{p}_s\rangle$ and $\langle\vec{q}\rangle$. It's easy to note that the momentum dependent part of x_{NW} is constant in time and consequently can only enlarge the clocks mechanism

dispersion D_0. Due to it the dispersion structure is the same as for nonrelativistic relative motion of (19) but its members are described by the more complicated formulaes omitted here. In fact this calculations evidence that x_{NW} meausurements introduces only additional time-independent clock dispersion of the order of G^2 Compton wavelength without changing our previous conclusions about time operator properties.

In fact F^1 proper time measurement in ARF can be performed by two different methods which equivalence must be proved. In the first method described above the detector D_0 installed in ARF measures τ_1 and induces C_x state collapse. In the second one the detector D_1 installed in F^1 measures the clock state and after it D_1 signal transfered to ARF. In this case we should consider the collapse in the moving frame , which is difficult to describe. But we must note that independently of its mechanism such interaction happens after this clocks evolves to this state and so can't influence directly on their evolution, so it seems correct to neglect it at this stage. Obtained time-fluctuation effect reminds the well-known life-time dilatation for the relativistic unstable particles [24]. In this framework such particle can be regarded as the elementary binary clock having only two states.

Obtained results evidence that the proper time in Quantum RF depend on the RF quantum state, but doesn't prove QEP directly. To do it we must consider two finite mass RFs on equal ground and to find the time transformation between them.

4 Relativistic Quantum Frames

To calculate the time operator between two RFs of finite mass it's necessary first to find the particle evolution equation in quantum RF rest frame. In general the system Poincare group irreducible representations contain the information which permit to describe its evolution completely, but due to appearance of time operators to find this representations for quantum RF is quite a problem. Therefore we choose another route; first we'll find the free particle evolution equation and corresponding proper time operator from Dirack constraints quantization. After it we'll investigate Poincare transformations for quantum RFs with the clues prompted by this Hamiltonian ansatz.

Dirack constraint formalism which permit to define free particle/antiparticle positive Hamiltonian was developed by Gitman and Tiutin [15]. They've shown that starting from free scalar particle action $S = -\int mds$ Dirack constraint quantization of $p_\mu^2 - m^2$ initial superhamiltonian results into positive square root Hamiltonian H_p as function of 3-momentum $\vec{p}$ plus additional charge $\xi = \pm 1$ discriminating antiparticles. In quantum case it was shown to result in Klein-Gordon square root (KGR) equation for both ξ

$$-i\frac{d\psi}{d\tau_0} = \sqrt{\vec{p}^2 + m^2}\psi$$

Following this approach we start from classical two particles action defined in ARF F^0 with time parameter τ_0 :

$$S(\tau_0) = \int L d\tau_0 = \int m_1[(\dot{x}_{10}^2 - \dot{x}_{1i}^2)^{\frac{1}{2}} + m_2(\dot{x}_{20}^2 - \dot{x}_{2i}^2)^{\frac{1}{2}}] d\tau_0$$

From it one finds 4-momentums $\pi_{1\mu}, \pi_{2\mu}$ which satisfy to superhamiltonian constraints $\pi^2_{j\mu} = m^2_j$. Due to m_1, m_2 dynamics independence S_2 system Dirack quantization results in double number of 3- momentums $\vec{p}_j$ and charges ξ_j. After simple calculations repeating Gitman-Tiutin anzats one obtains system Hamiltonian :

$$H_T = \sqrt{\vec{p}_1^2 + m_1^2} + \sqrt{\vec{p}_2^2 + m_2^2}$$

For two particles system quantization we can use this Hamiltonian in Schrodinger equation. But if we consider m_1 as quantum RF then like in nonrelativistic case, before quantize it we must put additional constraints corresponding to F^1 choice as rest frame and defining m_2 operators transformations into it. Analogously to nonrelativistic case we put constraint on F^1 momentum in its rest frame $\vec{p}_{11}^2 \approx 0$, meaning that RF don't move relative to itself. For charges we put constraint $\xi'_1 = \xi^2_1$ - .i.e. RF can't be antiparticle for itself, and correspondingly $\xi'_2 = \xi_1\xi_2$. Below for simplicity we'll describe in detail only restricted Hilbert space sector without antiparticles [15]. Corresponding to H_T two particles state vector isn't interpereted by us as their wave function in F^0. For two particles correlations it can result into contradictions connected with nonlocalities. But we don't need it in such role and will study only reduced state vector in quantum RF F^1.

To find the transformations from ARF to F^1 we consider the system S_2 of RF F^1 and particle G^2 which momentums $\vec{p}_i$, energies E_i are defined in ARF. For the constraint described above we choose ARF FC $\vec{p}_0^2 \approx 0$ and S_2 Hamitonian :

$$H^0_A = (m_0^2 + \vec{p}_0^2)^{\frac{1}{2}} + (m_1^2 + \vec{p}_1^2)^{\frac{1}{2}} + (m_2^2 + \vec{p}_2^2)^{\frac{1}{2}} \tag{27}$$

if G_2 is boson. Like in nonrelativistic case all $\vec{p}_i$ are the operators and state vector ascribed also to ARF $|\vec{p}_0 = 0\rangle$. This ARF constraint formalism reproduces all relativistic QM results for $m_0 \to \infty$. Performing transformations to F^1 rest frame we assume that the proper time parameter τ_1 can be defined in it from F^1 clocks measurements extrapolation as was described in previous chapter. Then from $\vec{p}_1$ constraint and correspondence with Lorentz momentum transformations we phenomenologically find m_i momentums:

$$\vec{p}_{12} = \vec{p}_i + \frac{(\vec{n}_1\vec{p}_i)(E_1 - m_1)\vec{n}_1 - E_i\vec{p}_1}{m_1} + \vec{F}_i(\vec{p}_0) \tag{28}$$

where $\vec{n}_1 = \vec{p}_1|\vec{p}_1|^{-1}$. $\vec{F}_i$ are undefined at this stage operators for which $\langle\vec{F}_i\rangle = 0$ and can be neglected in the following calculations. This transformation results in $\vec{p}_{11} = 0$ if $\vec{p}_0 = 0$ and will advocate its form below where we'll discuss Poincare group for quantum RFs. Until then this is phenomenological transformation which for definite RFs momentum and velocities reproduces Lorentz transformations. If G^2 have spin zero then the Hamiltonian H transformed from ARF to F^1 is equal :

$$H^1_{Tot} = H^1_0 + H^1_1 + H^1_2 = \sum_{i=0}^{2}(m_i^2 + \vec{p}_{1i}^2)^{\frac{1}{2}} \tag{29}$$

For classical Special Relativity where normally RF supposed to have infinite mass $\vec{p}_{1i}, H^1_R$ corresponds to the canonical momentums for finite mass RFs [25]. We see

that F^0 motion Hamiltonian is factorized and so we can drop it and regarding S_2 motion can use S_2 Hamiltonian $H^1 = H_1^1 + H_2^1$. In quantum case in H^1 we can't simply omit $\vec{p}_{11}$ because now it's operator. So in F^1 proper time τ_1 S_2 evolution equation is :

$$-i\frac{d\psi^1}{d\tau_1} = [(m_1^2 + \vec{p}_{11}^{\,2})^{\frac{1}{2}} + (m_2^2 + \vec{p}_{12}^{\,2})^{\frac{1}{2}}]\psi^1 = (s(\vec{q}) + \vec{p}_s^{\,2})^{\frac{1}{2}}\psi^1 \tag{30}$$

where S_2 c.m. observables $\vec{q}, \vec{p}_s$ defined in chap.3. Solutions of this equation describe G^2 normalized free wave packet localizable relative to F^1 rest frame :

$$\Psi^1(\tau_1) = \varphi_2(\vec{p}_{12})e^{-iE^1\tau_1}|m_2, \vec{p}_{12}\rangle|m_1, \vec{p}_{11} = 0\rangle = \varphi_2'(\vec{q})e^{-iE^1\tau_1}|\sqrt{s}, \vec{p}_s = \vec{p}_{12}\rangle|m_1, -\vec{q}\rangle|m_2, \vec{q}\rangle \tag{31}$$

expressed also via S_2 c.m. observables. Here $E^1 = E_1^1 + E_2^1$ are H^1 eigenvalues. They differ from the standard KGR energy only on m_1 and so we can use in F^1 rest frame the standard KGR momentum spectral decomposition and the states scalar product [11].

In F^1 rest frame together with its proper time τ_1 the space coordinate can be defined. We choose arbitrarily as G^2 coordinate (nonhermitian) operator in F^1 : $\hat{x}_{12} = i\frac{\partial}{\partial q_x}$ and corresponding Hermitian Newton-Wigner operator can be easily derived. Note that x_q defined in F^1 differs from the same operator defined in c.m.s., yet our following results doesn't depend on the particular form of this operator. x_{12} also differs from the operator $x_p = i\frac{\partial}{\partial p_{12x}}$ which corresponds to the classical distance between F^1 and G^2. They coincide only in the limit $m_1 \to \infty$ or in nonrelativistic case.

Now we can calculate F^2 proper time operator as function of the proper time in F^1. To perform it we assume again that F^2 c.m. motion is equivalent to the spinless particle G^2 motion. In the described framework the Hamiltonian of F^2 with C_0 or C_x clocks in F^1 rest frame can be obtained substituting in $\hat{H}^1$ of (30) $m_2 = m_2' + \hat{H}_c$. $\hat{\tau}_2$ can be found solving Heisenberg equation for F^2 clocks coordinate $\dot{x} = -i[x, H^1]$ analogously to (13). If we omit analogously to (23) the members describing the clocks mechanism fluctuations the F^2 proper time operator $\hat{\tau}_2$ is equal :

$$\hat{\tau}_2 = \frac{m_2\tau_1}{(m_2^2 + \vec{p}_{12}^{\,2})^{\frac{1}{2}}} \approx \frac{m_2'\tau_1}{(m_2'^2 + \vec{p}_{12}^{\,2})^{\frac{1}{2}}} = \hat{B}_1(\vec{p}_{12}, m_2')\tau_1 \tag{32}$$

This formalism is completely symmetrical and the operator obtained from (32) exchanging indexes 1 and 2 relates the time $\hat{\tau}_1$ in F^1 and F^2 proper time - parameter τ_2. The Special Relativity limit when τ_2 becomes the parameter is obvious and analoguosly to it the average time boost depends on whether F^1 measures F^2 clocks observables, as we consider or vice versa, and this measurement makes F^1 and F^2 nonequivalent [25]. The new effect will be found only when F^1 and F^2 will compare their initially synchronized clocks. In QM formalism this synchronization means that F^2 state prepared at the moment τ_0 can be factorized as $\Phi_2(\vec{p}_{12})\varphi_c(x, 0)$ analogous to (21). If this F^2 time measurements repeated several times (to perform quantum ensemble) it'll reveal not only classical Lorentz time boost , but also the

statistical spread having quantum origin with the dispersion given in (16). Obtained relation between two finite mass RFs proper times evidence that Quantum Equivalence principle can be correct also in relativistic case.

If the number of particles $N_g > 1$ then for the system state description the clasterization formalism can be used [21]. According to it for $N = 3$ Hamiltonian in F^1 of two free particles G^2, G^3 rewritten through the system canonical observables acquires the form :

$$\hat{H}^1 = (m_1^2 + \vec{p}_{11}^{\,2})^{\frac{1}{2}} + (s_{23} + \vec{p}_{1,23}^{\,2})^{\frac{1}{2}} = (s + \vec{p}_s^{\,2})^{\frac{1}{2}} \tag{33}$$

,where $\sqrt{s}_{23}$ is G^2, G^3 invariant mass, $\sqrt{s}, \vec{p}_s$ are the system total invariant mass and momentum. In clasterization formalism at the first level the relative motion of G^2, G^3 defined by $\vec{q}_{23}$ their relative momentum is considered. At the second level we regard them as the single quasiparticle - cluster C_{23} with mass $\sqrt{s}_{23}$ and momentum $\vec{q}$ in the system c.m.s. It transformed to $\vec{p}_{1,23}$ momentum in F^1 and so at any level we can regard the relative motion of two objects only. This procedure can be extended in the obvious inductive way to arbitrary N. If we have two reference frames F^1, F^2 and $N_g \neq 0$ then their relative momentums can be also described by the cluster formalism.

Due to appearance of the time operator between two RFs to find Poincare group transformations for quantum RFs $\hat{U}^s_{2,1}(\tau_2, \tau_1)$ is quite a problem and here we can present it only phenomenologically for some simple examples. We don't include rotations into consideration, so this results are suitable completely only for 1-dimensional case and for more dimensions they give only partial description of the Lorentz transformations. Consider first the case $N = 2$ when S_2 include F^1, F^2 only and its state in F^1 rest frame $\Psi^1(\tau_1)$ is the solution (31) of eq. (30). We'll take that it transformed by $U^F_{2,1}$ into state $\Psi^2(\tau_2)$ in F^2 rest frame. If F^1, F^2 clocks are synchronized at $\tau_1 = \tau_2 = 0$ then for this time moment $\Psi^2(0) = \hat{U}^F_{2,1}(0,0)\Psi^1(0)$ and from F^1, F^2 symmetry it follows : $|\Psi^2(0)\rangle = \varphi'_1(\vec{p}_{21})|m_2, \vec{p}_{22} = 0\rangle|m_1, \vec{p}_{21}\rangle$. $F^{1,2}$ internal wave functions $\varphi_c^{1,2}(x, 0)$ at $\tau_1 = 0$ are obviously invariant and so omitted here. Like in nonrelativistic case we introduce $\vec{p}_f = \vec{p}_{11} + \vec{p}_{12}$, $\vec{p}'_f = \vec{p}_{21} + \vec{p}_{22}$ and conjugated $\vec{r}_f, \vec{r}'_f$. From the correspondence with Lorentz transformations it should give $\langle\vec{p}_{12}\rangle = -\frac{m_2}{m_1}\langle\vec{p}_{21}\rangle$ and if to demand that fro relative momentum in F^2 $\vec{q}_2 = -\vec{q}$ must be fulfilled then the simplest transformation is :

$$\hat{U}^F_{2,1}(0,0) = P_f e^{i a_f \vec{p}_f \vec{r}_f} \tag{34}$$

where $a_f = \ln\frac{m_1}{m_2}$, P_f is $\vec{r}_f$ reflection (parity) operator. We see $\hat{U}^F_{21}(0,0)$ ansatz practically coincides with nonrelativistic transform of (5) for $N = 1$. The passive S_N transformation for spinless G^i also found from the correspondance principle as the minimal extension of standard Poincare transformations :

$$\hat{U}^s_{2,1}(0,0) = U^F_{21}(0,0)\prod_{j=3}^{N} e^{-i\vec{\beta}_f \vec{N}'_j} \tag{35}$$

where velocity operator $\vec{\beta}_f = \vec{p}_f (H_2^1)^{-1}$, $\vec{N}'_i = H_i^1 \frac{\partial}{\partial \vec{p}_{1i}} + \frac{\partial}{\partial \vec{p}_{1i}} H_i^1$ are G^i Poincare generators in F^1 which coincide with standard ansatz. Then the transformation

operator for arbitrary τ_1, τ_2 is :

$$\hat{U}^s_{21}(\tau_1,\tau_2) = \hat{W}_2(\tau_2)\hat{U}^s_{21}(0,0)\hat{W}_1^{-1}(\tau_1) \tag{36}$$

, where $\hat{W}_{1,2}(\tau_{1,2}) = exp(-i\tau_{1,2}\hat{H}^{1,2})$ are S_N evolution operators and $H^{1,2}$ - S_N Hamiltonians in F^1, F^2 rest frames.

It means that despite τ_2 and τ_1 are correlated only statistically through $\hat{\tau}_2$ nevertheless S_N state vectors for free motion in F^2, F^1 at this moments are related unambiguously. Transformed S_N momentums are :

$$\vec{p}_{21} = -\frac{m_2}{m_1}\vec{p}_{12} + d_1\vec{p}_{11} \quad , \vec{p}_{22} = -\vec{p}_{11} + d_2\vec{p}_{11} \tag{37}$$

$$\vec{p}_{2i} = \vec{p}_{1i} + \frac{(\vec{n}_{12}\vec{p}_{1i})(E^1_2 - m_2)\vec{n}_{12} - E^1_i\vec{p}_{12}}{m_2} + d_i\vec{p}_{11} \tag{38}$$

, where $\vec{n}_{12} = \frac{\vec{p}_{12}}{|p_{12}|}$, E^1_i are G^i energies in F^1. If to demand that all relative momentums $\vec{q}_{ij}$ conserved (or reflected), then d_i can be calculated, but due to their unimportance we omit it here. It's easy to see that $\vec{p}_{1i}$ of (28) for ARF to F^1 transform follows from $U^s_{2,1}$ after the simple substitutions, and so the semiqualitative Hamiltonian derivation of (28) was consistent. We see that the passive spinless G^3 transformation differs from the standard one only by the change of velocity parameter to the operator $\vec{\beta}$ which commutes with G^3 Hamiltonian.

It was argued that RF quantum properties can become important in Quantum Gravity , where in principle one should quantize the field, matter and RF simultaneously [5, 6, 26]. In principle our approach permits to calculate the time operator $\hat{\tau}_1$ for RF F^1 moving in the external gravitational field $g_{\mu\nu}(x)$. We assume that ARF is located in the region where this field is weak and so we can take $\tau_0 = x_0$ - world time. Analogously to (12) F^1 clocks Hamiltonian in ARF (for $g_{oa} = 0$ gauge) :

$$\hat{H}_T = [g_{00}(m_1 + H_c)^2 + g_{00}g_{ab}p^a_1p^b_1]^{\frac{1}{2}} \tag{39}$$

,where $a, b = 1, 3$ [25]. Now H_T depends on x_μ and due to it solving Heisenberg equation (13) for the clocks hand coordinate x_c one obtains the differential relation for τ_1 :

$$d\hat{\tau}_1 = \frac{\sqrt{g_{00}}(m_1 + H_c)d\tau_0}{[(m_1 + H_c)^2 + g_{ab}p^a_1p^b_1]^{\frac{1}{2}}} = \sqrt{g_{00}}B_g(x, \vec{p}_1)d\tau_0 \tag{40}$$

In this case $\hat{\tau}_1$ becomes the integral operator , where integral is taken over τ_0 interval. If $g_{\mu\nu}$ is the classical metrics then this relation contains no new physics , except the additional gravitational 'red shift' time boost proportional to $\sqrt{g_{00}}$ [25]. But in Quantum Gravity $g_{\mu\nu}(x)$ becomes the operator and its fluctuations can induce the additional quantum fluctuations of the measured F^1 clocks time. Despite that this fluctuation calculations are quite complicated we can expect from the general Quantum Statistics rules [17] that they can be factorized from the considered Lorentz boost fluctuations induced by the F^1 momentum fluctuations :

$$D_T = D_G(\tau_0) + D_L(\tau_0) + D_o(\tau_0)$$

From this rules we can expect also that for F^1 motion in the homogeneous gravitation field D_G will grows proportionally to τ_0 analogous to QED fluctuations (Brownian motion effects). Note that this fluctuations must be independent of RF mass.

This approach can give some new insight into the famous time problem of Quantum Gravity [5, 26] which we discuss here briefly. In this aspect the situation in Classical and Quantum Gravity seems to differ principally. Strictly speaking if the metrics becomes the operator it stops to be space-time metrics which unambiguously defines the space-time geometry. Due to it the observer can correctly use only the operational definition of physical space-time by means of clocks and other measurements. In gravity this operational time can originate from some evolving observable of gravitation field or to be the operator describing the time measurement for some free matter object carrying some nongravitational 'foreign' clocks. The idea that space-time events can be described by their relation with some distributed system or media was extensively explored for long time [7] . The most close to our purposes is the incoherent dust system, each piece of it carrying clocks. Gravity ADM quantization for such system with selfgravitation account permits to extract positive Schrodinger hamiltonian as was shown by Brown and Kuchar [26]. Hence the dust pieces motion in their model was described only semiclassically. Introduction of 'dust space' $\vec{z}$ permit to quantize the gravitation field. Yet the free quantum motion of dust pieces transforms $\vec{z}$ into the operator on the initial space-time manifold x_μ which makes this quantization procedure contradictory.

We describe here briefly the model of dust RFs quantum motion where in the first approximatio its selfgravitation neglected. Let's consider first classical RF F^1 free falling in external gravitational field. In F^1 comoving 'Gaussian' frame where frame conditions imposed before the field variation we have $g'_{00} = 1, g'_{0a} = 0$. In this RF for the classical field gravity constraints fulfilled $H_a(x) = 0, H_0(x) = 0$ which permit to calculate g'_{ab}, p'_{ab} evolution for F^1 clock time solving corresponding Hamilton equations for H_0 [26]. In quantum case this vacuum field constraints results in Wheeler - deWitt equation $\hat{H}_0\Psi = 0$ from which Schrodinger Hamiltonian can't be derived easily. Now let's account RF quantum motion and suppose that this constraints holds true also in quantum F^1 comoving frame. F^1 proper time for the external observer is given by the operator analogous to (40), but in comoving frame τ_1 is just the parameter. In this case we can calculate field observables evolution in F^1 clocks time from Heisenberg equations for H_0 vacuum constraint :

$$\dot{g}'_{ab}(x) = -i[g'_{ab}(x), H_0(x)]$$

where the commutator in general is nonzero. Note that this equation is obviously local, so to calculate $g'_{ab}(x, \tau_1)$ we must define g'_{ab}, p'_{ab} only on a small spacelike surface region around x at a preceding moment $\tau_1 - d\tau_1$. Space coordinates x_a supposedly can be defined at least in the close vicinity of F^1 analoguosly to the definition given above for the flat space-time. Obviously this approach have many associated problems some of which are the construction of multifingered time for quantum RF dust and the field theoretical behavior of such commutators, despite it seems to deserve additional study.

For the conclusion we can claim that the extrapolation of QM laws on free

macroscopic objects regarded as RFs prompt to change the common approach to the space-time which was taken copiously from Classical Physics. In this paper the relativistic covariant theory of quantum RFs constructed and at least in flat space-time it agrees with the principle of equivalence for quantum RFs. The quantum RF momentum uncertainty results in the quantum statistical fluctuations of Lorentz boost which relates the proper times in two RFs. So in this model each observer has its proper time - parameter and euclidian coordinate space which can't be related unambiguously with the another observers space-time and in this sense is local.

References

[1] Y.Aharonov, T.Kaufherr Phys. Rev. D30 ,368 (1984)

[2] S.Doplicher et. al. Comm. Math. Phys. 172,187 (1995)

[3] L.I.Schiff, 'Quantum Mechanics' (New-York, Macgraw-Hill,1955)

[4] M. Toller , gr-qc 9605052

[5] C.Rovelli, Class. Quant. Grav. 8, 317,(1991)

[6] W.G.Unruh, R.M.Walde Phys. Rev. D40, 2598, (1989) , H.Kitada Nuov. Cim. 109B , 281 (1995) , gr-qc/9708055

[7] B. DeWitt , 'Gravitation : an Introduction to Current Research' ed. L. Witten , (Wiley, N-Y, 1962)

[8] S.N. Mayburov , in 'Proc. of 6th Quantum Gravity Seminar', Moscow, 1995 (W.S.,Singapore,1997) ,Proc. of 8th Marsel Grossman Conference', Jerusalem, 1997 (W.S., Singapore, 1998) , eprint gr-qc 9705127 , quant-ph/9801075

[9] S.N.Mayburov, Int. Journ. Theor. Phys. 34,1587 (1995) ,ibid. 37, 401 (1998)

[10] D'Espagnat Found. Phys. 20,1157,(1990)

[11] S. Schweber 'An Introduction to Relativistic Quantum Field Theory' ,(New-York,Row-Peterson ,1961)

[12] P.A.M.Dirack 'Lectures on Quantum Mechanics' , (New-York, Yeshiva University ,1964)

[13] D.Gitman , I.Tyutin 'Canonical Quantization of Fields with Constraints', (Berlin, Springer, 1990)

[14] A.Barut, R.Ronczka 'Theory of Group Representations and Applications', (PWN,Warszawa,1977)

[15] D.Gitman , I.Tyutin Class. Quant. Grav. 7 ,2131, (1990), JETP Letters 51,4(1990)

[16] A.Peres Am. Journ. Phys. 48, 552 (1980)

[17] A.Holevo 'Probabilistic and Statistical Aspects of Quantum Theory', (North-Holland,Amsterdam,1982)

[18] V.Fock Sow. Phys. 12, 404, (1937)

[19] E.Wigner, T.Newton Rev. Mod. Phys. 21,400,(1949)

[20] Y.Aharonov, D.Albert Phys. Rev. D 24 ,359 (1981)

[21] F.Coester Helv. Phys. Acta 38, 7 (1965)

[22] E.Byckling, K.Kajantie 'Particles Kinematics' (London, John Wiley and Sons ,1973)

[23] L.Landau, E.Lifshitz 'Classical Field Theory' (Moscow , Nauka ,1981)

[24] E.Byckling, K.Kajantie 'Particles Kinematics' (London, John Wiley and Sons ,1973)

[25] L.Landau , E.Lifshitz 'Statistical Physics' , (Moscow , Nauka ,1979)
(Moscow ,Nauka ,1981)

[26] J.Brown K.Kuchar Phys. Rev. D51 ,5600 (1995); ibid D43, 419 (1991)